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# IMAGE FUSION USING HIGHER ORDER SINGULAR VALUE DECOMPOSITION

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## ABSTRACT

The goal of image fusion is to combine relevant information from two or more source images into one single image such that the single image contains as much information from all the source images as possible. There are many image fusion methods. This paper present some of the image fusion techniques for image fusion and propose novel higher order singular value decomposition (HOSVD) based image fusion algorithm. Image fusion depends on local information of source images, the proposed algorithm picks out informative image patches of source images to constitute the fused image by processing the divided subtensors rather than the whole tensor. The sum of absolute values of the coefficients (SAVC) from HOSVD of subtensors is employed for activity-level measurement to evaluate the quality of the related image patch, and a novel sigmoid-function-like coefficient-combining scheme is applied to construct the fused result.

**KEYWORDS:** Image fusion, Discrete cosine transform, Discrete wavelet transform, higher order singular value decomposition (HOSVD), sigmoid function.

## INTRODUCTION

Fusion can be described as the process of combining two or more different entities to form a new entity. Therefore, Image fusion is the process of combining two or more distinct images to form a new single image which will be better and more informative than every other input image. With the progress in technology, we can now obtain information from images of different sources to produce a new high quality image which also contains spatial and spectral information [1]. Thus, Image Fusion can be described as a process that improves the quality of information of a set of images. [1]-[5]. Image fusion find application in the area of navigation guidance, object detection and recognition, medical diagnosis, satellite imaging for remote sensing, rob vision, military and civilian surveillance, etc. Image fusion systems are widely used in surveillance and navigation applications, for both military and domestic purposes [5], for example, Due to the limited depth-of-focus of optical lenses in CCD devices, it is often not possible to get an image that contains all relevant objects "in focus".

To achieve all objects "in focus", a fusion process is required so that we get resultant image with all objects in focus.

Image fusion methods can be broadly classified into two groups - spatial domain fusion and transform domain fusion. The fusion methods such as averaging, Brovey method, principal component analysis (PCA) and IHS based methods fall under spatial domain approaches. The disadvantage of spatial domain approaches is that they produce spatial distortion in the fused image. Spatial distortion can be very well handled by frequency domain approaches on image fusion.

Laplacian pyramid [1], , Discrete Wavelet Transform (DWT) [3], Discrete Cosine Transform (DCT) [4] etc., image fusion methods comes under transform domain. These methods show a better performance in spatial and spectral quality of the fused image compared to other spatial methods of fusion. These transform domain based methods merge the transform coefficients using the classical weighted average strategy or the choose-max strategy and then obtain the fused result through the inverse transformation of the combined coefficients. A novel HOSVD-based image fusion, constructed multiple input images as a tensor and can evaluate the quality of image patches using HOSVD of subtensors. Then, it employed a novel sigmoid-function-like coefficient - combining scheme to obtain the fused result [9]. A tensor is a multidimensional array. More formally, an N-way or Nth-order tensor is an element of the tensor product of N vector spaces, each of which has its own coordinate system. Tensor-based information processing methods are more suitable for representing high-dimensional data and extracting relevant information than vector- and matrix based methods and thus receives lots of attention [6]-[8]. As one of most efficient tensor decomposition techniques, higher order singular value decomposition (HOSVD). Motivated by the salient ability of HOSVD to represent high-dimensional data and extract features, this paper proposes a novel HOSVD based image fusion algorithm [7]. It is worthwhile to highlight several aspects of the proposed transform domain-based approach here.

First , consider two or multiple source images of the same scene and are somewhat similar i.e. the same physical structures in the environment. Fusion algorithms are input dependent . If source images do not have same physical structure in environment then solution is to preprocess the source images. One of the important pre-processing steps for the fusion process is image registration. After getting source images with same structure as require by proposed algorithm, construct them in to a tensor and employs the HOSVD technique to extract their features simultaneously. This paper picks out image patches which contains maximum information of source images to constitute the fused image by processing the divided subtensors rather than the whole tensor.

A slice of the core tensor results from HOSVD of subtensors reflects the quality of the related image patch. Unlike the conventional activity-level measurements, which apply the absolute value of a single coefficient to evaluate the corresponding pixel, paper employs the sum of absolute values of coefficients (SAVC) as the activity-level measurement of the related patch. To adapt to different activity-level measurements (approximate or substantially different), propose a novel and flexible sigmoid-function-like coefficient-combining scheme, which incorporates the usual choose-max scheme and the weighted average scheme [9].

## DISCRETE WAVELET TRANSFORM BASED FUSION

The discrete wavelets transform (DWT) allows the image decomposition in different kinds of coefficients preserving the image information. Such coefficients coming from different images can be appropriately combined to obtain new coefficients, so that the information in the original images is collected appropriately. Once the coefficients are merged, the final fused image is achieved through the inverse discrete wavelets transform (IDWT), where the information in the merged coefficients is also preserved [3].

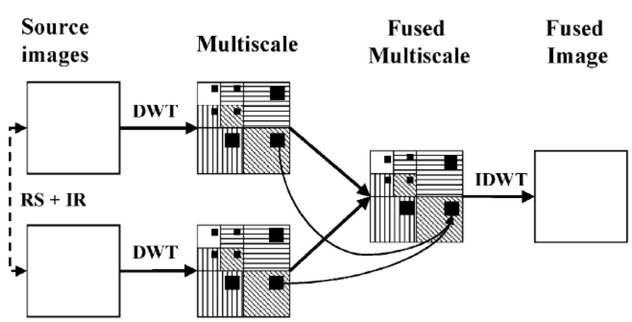


*Fig. 1. Block diagrams of generic fusion schemes where the input images have identical [3]*

The DWT is applied to both source images and a decomposition of each original image is achieved. The different black boxes as shown in fig.1, associated to each decomposition level, are coefficient corresponding to the same image spatial representation in each original image, i.e. the same pixel or pixels positions in the original images. Only coefficients of the same level and representation are to be fused, so that the fused multiscale coefficients can be obtained. This is displayed in the diagonal details where the curved arrows indicate that both coefficients are merged to obtain the new fused multiscale coefficient. This is applicable to the remainder coefficient. Once the fused multiscale is obtained, through the IDWT, the final fused image is achieved [3].

## DISCRETE COSINE TRANSFORM BASED FUSION

This paper studies image fusion in the DCT domain, also, present an image fusion technique based on a contrast measure defined in the DCT domain in JPEG framework. This technic is faster than the wavelet based image fusion

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technique when the images to be fused were saved in JPEG format or when the fused image will be saved or transmitted in JPEG format. There is no difference in visual quality between the fused image obtained by image fusion technique based on a contrast measure defined in the DCT domain and that obtained by the wavelet transform based image fusion technique.

In this algorithm first, divides up the original images into 8 by 8 pixel blocks, and then calculates the discrete cosine transform (DCT) of each block. A quantizer rounds off the DCT coefficients according to the quantization matrix. Quantization of the DCT coefficients is a lossy process. Then entropy coding is used to encode the quantized coefficients and a compression data stream is output. In the decoder, JPEG recovers the quantized DCT coefficients from the compressed data stream, takes the inverse DCT transform and displays the image. Here we use different fusion techniques to obtain the fused images

The key step is to fuse the DCT representations of multi-images into a single DCT representation of the fused image. A contrast sensitivity method is adopted to produce a visually better fused image. This is based on the fact that the human visual system is sensitive to local contrast [4].

## MATHEMATICAL BACKGROUND OF HOSVD

**Singular Value Decomposition**

Every matrix $A \in R^{m\times n}$ can be written as the factorisation of three matrices U, $\sum$ and V , where

$$A = U \sum V^T, \sum = diag(\sigma_1 \ldots\ldots \sigma_p) \in R^{m\times n}, p = \min\{m, n\} \quad (1)$$

with $U \in R^{m\times m}$ and $V \in R^{n\times n}$ being orthogonal matrices and $\sigma_1 \geq \sigma_2 \geq \cdot \ldots\ldots \geq \sigma_p \geq 0$.

- $\sigma_1 \ldots\ldots \sigma_p$ are the singular values of A.
- The columns $u_1 \ldots\ldots\ldots u_m$ are called left singular vectors of A.
- The columns $v_1 \ldots\ldots\ldots v_n$ are called right singular vectors of A.

Notice that $\sum$ is a pseudo diagonal matrix.
When the SVD of a matrix $A \in R^{m\times n}$ is computed, then we can keep only the first n rows of $\sum$ so that we have a diagonal matrix. Of course, U has to be modified accordingly. In this case we set

$$A_{red} = U_{red} \sum{}_{red} V_{red}^T$$

where
$U_{red} = U[:, 1 : n]$, $\sum_{red} = \sum(1 : n, 1 : n)$
This trimmed down version of the SVD is called thin SVD [GL96]. The thin SVD will be relevant for the chapter about Latent Semantic Analysis, where it is used to detect the latent semantics of the stored two-dimensional structure.
The SVD is related to the eigen-decomposition of $AA^T \in R^{m\times m}$ and $A^T A \in R^{n\times n}$, since $A^T A v_i = \sigma_i^2 v_i$ and $AA^T u_i = \sigma_i^2 u_i$ [SA03].
Some important properties of the SVD are the following:

- The singular values of a matrix are uniquely determined.
- $A = U \sum V^T \equiv A^T = U \sum^T V^T$
- If rank(A) = r, then A has r nonzero singular values $\sigma_1 \geq \sigma_2 \geq \cdot \ldots\ldots \geq \sigma_r > 0$

The definitions and properties discussed above extend to complex matrices.
The SVD has an important role to play in linear algebra, being used for pseudoinverse computing, least square problems, computation of the Jordan canonical form or solving integral equations, just to mention a few.

**Tensor**

Tensor. A tensor (also called multidimensional array or n-way array) is the generalisation of a matrix. A matrix can be seen as a two-dimensional tensor, a vector as a one-dimensional tensor. We refer to the di_erent dimensions of a given tensor as its modes.
Example: the third mode of a tensor $A \in R^{I\times J\times K}$ is of size K.
**Tensor order**. The order of a tensor $A \in R^{I_1 \times \ldots\ldots \times I_N}$ is N. Its elements are denoted as $a_{i_1 \ldots\ldots i_n \ldots i_N}$ where $1 \leq i_n \leq I_n$ for $1 \leq n \leq N$.

**Norm of a tensor**. The norm of a tensor A is defined by the Frobenius-norm of any of its existing unfoldings.

$$\|A\| := \|A_{(1)}\|_F = \cdots = \|A_{(N)}\|_F \qquad (2)$$

The Frobenius-norm of a complex matrix $A \in C^{m\times n}$ is given by

$$\|A\|_F := \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2} \qquad (3)$$

**Rank of a tensor**. The n-rank of A, $rank_n(A)$, is the dimension of the vector space spanned by the n-mode vectors and the following equation holds:

$$rank_n(A_{(n)}) = rank(A) \qquad (4)$$

**Tensor product**. The tensor product (denoted by the operator $\otimes$) for tensors $A \in R^{d_1\times\dots\dots\times d_N}$, $B \in R^{f_1\times\dots\dots\times f_M}$ is defined as:

$$c_{i_1\dots\dots,i_N,j_1\dots\dots j_M} := a_{i_1\dots\dots,i_N,}b_{j_1\dots\dots,j_M,} \qquad (5)$$

where

$$C = A \otimes B \in R^{d_1\times\dots\dots\times d_N\times f_1\times\dots\dots\times f_M}$$

**n-mode product**. Let $A \in R^{I_1\times\dots\dots\times I_N}$ and $M \in R^{J_n\times I_n}$ be given.

Then $A \times_n M \in R^{\prod_{i=1}^{n-1} I_i\times J_n\times\prod_{i=n+1}^{N} I_i}$, and we define the new tensor elementwise as

$$(A \times_n M)_{i_1\dots i_{n-1}j_n i_{n+1}\dots i_N} := \sum_{i_n\in I_n} a_{i_n\dots\dots I_n}\, m_{j_n\, i_n} \qquad (6)$$

The operator $\times_n$ is called the n-mode product.

**Higher-Order Singular Value Decomposition**.

The Higher-Order Singular Value Decomposition (HO-SVD) of a tensor $A \in R^{I_1\times\dots\dots\times I_N}$ is defined as

$$A = S \times_1 U_1 \times_2 \dots\dots \times_N U_N \qquad (7)$$

when the following properties hold:

- The $U_i$ are orthogonal for $i \in \{1, \dots, N\}$. These matrices are composed by the left singular vectors of the corresponding matrix-unfoldings of A.
- The core tensor $S \in R^{I_1\times\dots\dots\times I_N}$, and all subtensors with fixed index are all- orthogonal, which means $\langle S_{i_n=0}, S_{i_n=0}\rangle = 0$ for all $a, b \in \{1, \dots, N\}$ where a≠b.
- $S \in R^{I_1\times\dots\dots\times I_N}$, and all subtensors with fixed index are ordered ($\|S_{i_n=1}\| \geq \cdots \geq \|S_{i_n=I_n}\| \geq 0$ for all possible values of n). Notice the similar properties of S in the HO-SVD and $\sum$ in the SVD.
- $\sigma_{n,i} := \|S_{i_n=1}\|$ are called n-mode singular values and the vectors $u_{n,i} \in R^n$ are called n-mode singular vectors of A, analogously to the SVD definitions.

Decompositions of higher-order tensors have useful applications in many different fields. The HO-SVD is used in psychometrics, chemometrics, signal processing, compression algorithms, numerical linear algebra and analysis, statistics, computer vision, OCR, data mining, personalised web-search, neuroscience and graph analysis, just to mention a few.

There is an important link between HO-SVD and SVD :

"Let $A = S \times_1 U_1 \times_2 \dots\dots \times_N U_N$ a HO-SVD, then the SVD of the n-mode matrix unfolding $A_{(n)}$ is: $A_{(n)} = U_n \sum_n V_n^T$, with $\sum_n = diag(\sigma_{n,1}, \dots, \sigma_n I_n) \in R^{I_n\times I_n}$ and,

$$V_n^T := (\textstyle\sum_n^{-1} S_n) \cdot \left(U_{n+1} \otimes \dots \otimes U_N \otimes U_1 \otimes \dots \otimes U_{n-1}\right) \cdot \in R^{I_{n+1}\dots I_1 I_N \dots I_{n-1}\times I_n}$$

Where $\otimes$ denotes the Kronecker product".

This means that we can compute the HO-SVD of a n-mode tensor A by computing the SVD of its first n tensor unfoldings. In fact, all known methods for computing the HO-SVD of a tensor need the SVD of its unfoldings. The complete SVD of the unfoldings is actually not needed, but only the matrices containing the left singular vectors in order to compute the approximation of the tensor.

The Kronecker product of two matrices $B \in R^{m\times n}$ and $C \in R^{p\times q}$ is given by:

$$A \otimes B = \begin{pmatrix} b_{11}C & b_{12}C & \dots & b_{1n}C \\ b_{21}C & b_{22}C & \dots & b_{2n}C \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1}C & b_{m2}C & \cdots & b_{mn}C \end{pmatrix} \in R^{mp\times nq}$$

Here, we introduce several notations and operations of tensors, which will be used in the rest of this paper (see [8] for details).

1. An $N^{th}$ order tensor is an object with N indices, i.e., $A \in R^{I_1 \times I_2 \times \dots \dots \times I_N}$.

2. An $n^{th-}$ mode vector of an $(I_1 \times I_1 \times \dots \times I_N)$- dimensional tensor A is an $I_n$ – dimensional vector obtained by varying index $i_n$ but fixing the indices.
3. The $n^{th-}$ mode product of a tensor $A \in R^{I_1 \times I_2 \times \dots \dots \times I_N}$ and a matrix $U \in R^{J_n \times I_n}$ along the $n^{th}$ mode is denoted by

   B = $U \in R^{I_1 \times I_2 \times \dots \dots \times I_{n-1} \times J_n \times I_{n+1} \times \dots \times I_N}$ with elements $b_{i_1, i_2 \times \dots \dots \times i_{n-1}, j_n, i_{n+1} \times \dots \times i_N} = \sum_{i_n=1}^{I_n} a_{i_1, i_2 \times \dots \dots \times i_{n-1}, j_n, i_{n+1} \times \dots \times i_N} \bullet u_{j_n, i_n}$, where $u_{j_n, i_n}$ stands for the $(j_n, i_n)^{th}$ element of matrix U, and $a_{i_1, i_2 \times \dots \dots \times i_{n-1}, j_n, i_{n+1} \times \dots \times i_N}$ represents the $(i_1, i_2 \times \dots \dots \dots \times i_{n-1}, \; i_n, i_{n+1} \times \dots \dots \times i_N)$th element of tensor A.
4. The $n^{th-}$ mode matricization of a tensor A is an operation where the $n^{th}$ – mode vectors of A are aligned as the columns of a matrix which is denoted by $A_{(u)}$.
5. HOSVD of tensor $A \in R^{I_1 \times I_2 \times \dots \dots \times I_N}$ is given by $A = \sum \times_1 U_1 \times_2 U_2 \dots \dots \times_N U_N$, where $\sum \in R^{I_1 \times I_2 \times \dots \dots \times I_N}$ is the core tensor that satisfies the all-orthogonality conditions, and $U_n \in R^{J_n \times I_n}, n = 1,2, \dots \dots \dots N$, are the left singular vectors of $A_{(u)}$.

## HOSVD BASED FUSION

### Description of Proposed Algorithm

Generally, a transform-domain fusion algorithm consists of the following three steps: 1) obtain the decomposition coefficients using some transform; 2) construct the activity-level measurement from these coefficients; and 3) merge these coefficients to construct the fused result in line with the measurements above. In the remainder of this section, a new image fusion algorithm is developed according to the steps above. HOSVD is one of most efficient data-driven decomposition techniques and can extract the features of multiple slices of the decomposed tensor simultaneously. To facilitate the description, we begin with two (M × N) - dimensional gray images [9].

**Step 1**: Two source images are constructed into a tensor with (M × N× 2) - dimensions (i.e., with three modes: the row, the column, and the label of the source image order), and HOSVD is employed to extract the related features (i.e., to obtain the decomposition coefficients). Although HOSVD is used to obtain the decomposition coefficients (or extract features) of multiple images, there are two important differences. we form $(\breve{M} \times \breve{N} \times 2)$- dimensional subtensors $A_i$, using two image patches $B_i(1)$ and $B_i(2)$ separately from the two source images and perform the HOSVD of $A_i$ , so that informative image patches are picked out to piece together the final fused image. Use the $n^{th}$-mode product of the core tensor and the third-mode factor matrix to reflect the quality of the related image patch for the purpose of constructing the final fused result from the product above more conveniently [8].

**Step 2**: It is commonly thought that the magnitude (absolute value) of the decomposed coefficient is consistent with the related local energy, which implies that the larger the absolute value of the coefficient is, the more information the corresponding pixel contains. Therefore, many transform domain fusion methods employ the absolute value of the coefficient as the activity-level measurement of the corresponding pixel. Borrowing the idea but unlike it, defines the SAVC as the activity-level measurement of the related image patch to evaluate its quality [9].

**Step 3**: To derive the coefficient-combining scheme, we first consider all possibilities: 1) In the same subtensor, the image patch with an even higher SAVC value contains more rich information or is of higher quality; thus, it should be directly selected as the final fused result of the corresponding subtensor (i.e., in this case, the choose-max strategy should be applied). 2) If the SAVCs of both image patches are close to each other, then they have approximate image quality, and thus, their weighted average should be used as the ultimate fused result of the subtensor (i.e., in this case, the weighted average strategy should be employed).

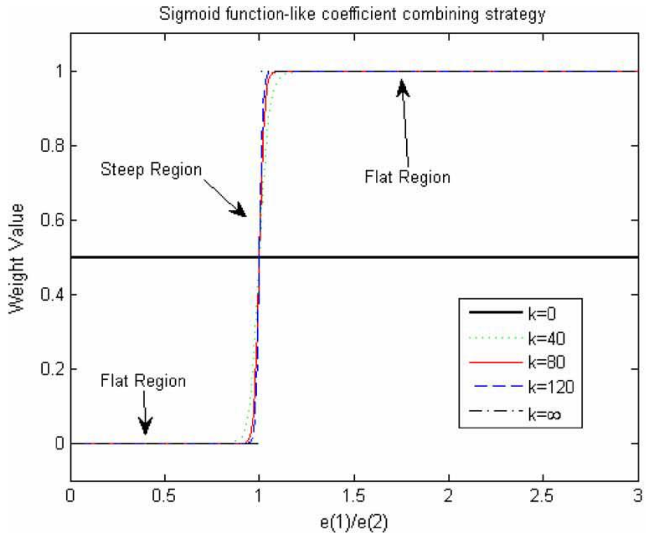


***Fig. 2. Sigmoid function with different shrink factors [9]***

However, for the first case, when two adjacent image patches are chosen, respectively, from different source images, it will cause discontinuous gap pixels between adjacent patches of the fused image. Therefore, a choose-max strategy with the smoothing function should be designed. In order to attain the aim above, this paper designs a novel sigmoid-function-like coefficient-combining scheme to adapt to different cases: the first situation is mapped into the flat region [marked in Fig.2] of the sigmoid function's range, and then, the approximate choose-max scheme works. The second one is projected into its steep region [marked in Fig.2], and in this case, the weighted average scheme works [9]. Thus, the proposed algorithm is summarized here.

1) Initialization

Construct two (M × N)-dimensional source images into a tensor with (M × N× 2)-dimensions. To further avoid the discontinuous gap above, the consecutive subtensors are enabled to partly share data, i.e., a sliding window technique is applied here to divide the tensor into ($\breve{M}$ × $\breve{N}$× 2)-dimensional subtensors with moving step size $p$ , which satisfies with $p \leq \breve{M}$ and $p \leq \breve{N}$. Note that I= fix ((M - $\breve{M}$ +1) / $p$) • fix ((N -$\breve{N}$ +1 ) / $p$), where ((M - $\breve{M}$ +1) / $p$) stands for the nearest integers (toward zero) of ((M - $\breve{M}$ +1) / $p$).

2) For $i = 1, 2, \ldots I$, let the HOSVD of divided subtensor $A_i$ be given by

$$A_i = \Sigma_i \times_1 U_i \times_2 V_i \times_3 W_i \quad (8)$$

To construct the fused result conveniently, we employ the following tensor:

$$\bar{\Sigma}_i = \Sigma_i \times_3 W_i \quad (9)$$

as the features of image patches rather than the original core tensor $\Sigma_i$ Based on $\bar{\Sigma}_i$ , each image patch $B_i(l)$, l = 1,2,…. $\breve{N}$,of subtensor $A_i$ can be represented as

$$\mathbf{B}_i(l) = \mathbf{U}_i \times \bar{\Sigma}_i(:, :. l) \bullet \boldsymbol{V}_i^T \ , \ l = 1,2. \quad (10)$$

To facilitate the description, $\mathbf{B}_i(l)$ is lexicographically ordered as $vec(\mathbf{B}_i(l))$ in a vector form, i.e.,

$$vec(\mathbf{B}_i(l)) = \sum_{m=1}^{\breve{M}} \sum_{n=1}^{\breve{N}} \bar{\Sigma}i\,(m,n,l) \ \bullet vec(\ \mathbf{u}m \ \times v_i^T) \quad (11)$$

where $\mathbf{u}_m$ represents the m^th^ column of $\mathbf{U}_i$ and $v_n$ stands for the nth column of $V_i$. Since both $\mathbf{U}_i$ and $V_i$ are orthogonal matrices, $vec(\mathbf{u}_m \times v_i^T)$, m = 1,2,….. $\breve{M}$ , n = 1,2, ,…. $\breve{N}$, , form an orthogonal basis, which implies that element $\bar{\Sigma}_i$(m, n, l) is actually the projection coefficient of $vec(\mathbf{B}_i(l))$ on $vec(\mathbf{u}_m \times v_i^T)$

Based on coefficient matrix $\bar{\Sigma}_i$ (**:, :.** l) the activity-level measurement of image patch $B_i(l)$ is defined as

$$e_i\ (l) = \sum_{m=1}^{\breve{M}} \sum_{n=1}^{\breve{N}} \left|\bar{\Sigma}i\,(m,n,l)\right|, \ \ l = 1,2 \quad (12)$$

which can be represented in another form as

$e_i(l) = \left\| vec(\overline{\Sigma}_i(:,:.l)) \right\|_1$, l = 1,2 (13) i.e., the activity-level measurement is the p1 norm of vector $vec(\overline{\Sigma}_i(:,:.l))$.

According to these activity-level measurements $e_i$(l), l = 1, 2, coefficient matrices $\overline{\Sigma}_i(:,:.1)$ and $\overline{\Sigma}_i(:,:.2)$ are merged to obtaina new coefficient matrix $D_i$, i.e.,

$$D_i = \frac{1}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} \times \overline{\Sigma}_i(:,:.1) + \frac{\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} \times \overline{\Sigma}_i(:,:.2) \qquad (14)$$

Where k is the shrink factor of the sigmoid function.

After $D_i$ is obtained, fused image patch $F_i$, is determined as follows:

$F_i = U_i \times D_i \times V_i^T$, I = 1, 2, ….I (15) 3) Finally, fused imageis constructed with, $F_i$, i = 1, 2,. .I: a) Initialize G as a zero matrix, i.e.,G = $0_{M\times N}$ ; b) superimposed $F_i$ onto G at its crresponding patch position, i = 1, 2, …..I; and c) for each pixel position of G, the added pixel value divided by its adding times is employed as the final fused result of this position.

**Discussion of the Sigmoid Function**

First, we consider one of two limit cases, i.e. k = + ∞, .If ($e_i(1)$ / $e_i(2)$) > 1, then

$$\frac{1}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} = 1$$

$$\frac{\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} = 0$$

Otherwise, If ($e_i(1)$ / $e_i(2)$) < 1 if, then

$$\frac{1}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} = 0$$

$$\frac{\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)} = 1$$

Obviously, the proposed coefficient-combining strategy reduces into the choose-max strategy. In other words, the choose-max scheme is just the special case of the proposed coefficient-combining strategy, i.e.,

$$F_i = U_i \times D_i \times V_i^T$$

$$= \begin{cases} U_i \times \overline{\Sigma}_i(:,:.1) \times V_i^T & \text{if } ei(1) > ei(2) \\ U_i \times \left(\frac{1}{2}\overline{\Sigma}_i(:,:.1) + \frac{1}{2}\overline{\Sigma}_i(:,:.2)\right) \times V_i^T & \text{otherwise} \\ U_i \times \overline{\Sigma}_i(:,:.1) \times V_i^T & \text{if } ei(2) > ei(1) \end{cases}$$

$$= \begin{cases} B_i(1) & \text{if } ei(1) > ei(2) \\ \frac{1}{2}(B_i(1) + B_i(2)) & \text{otherwise} \\ B_i(2) & \text{if } ei(2) > ei(1) \end{cases} \qquad (16)$$

Then, we consider another limit case, i.e., k = 0. In this case

$$\frac{1}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)}=\frac{1}{2}$$

$$\frac{\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)}{1+\exp\left(-k\ln\left(\frac{e_i(1)}{e_i(2)}\right)\right)}=\frac{1}{2}$$

Thus, the proposed algorithm is reduced into the average fusion method, i.e.

$$\begin{aligned} F_i &= U_i \times D_i \times V_i^T \\ &= U_i \times \left(\frac{1}{2}\overline{\Sigma}_i(:,:.1)+\frac{1}{2}\,\overline{\Sigma}_i(:,:.2)\right)\times V_i^T \\ &= \frac{1}{2}(B_i(1)+B_i(2)) \qquad (17) \end{aligned}$$

To facilitate the analyses, we plot the sigmoid function

$$\frac{1}{1+\exp\left(-k\ln\left(\frac{e(1)}{e(2)}\right)\right)}$$

with different in Figure.4.1, From this figure, several aspects can be observed

1) As k increases

$$\frac{1}{1+\exp\left(-k\ln\left(\frac{e(1)}{e(2)}\right)\right)}$$

approaches

$\frac{1}{2}+\frac{1}{2}\,\mathrm{sgn}\left(\frac{e(1)}{e(2)}-1\right)$

Where sgn (•) is the sign function. In particular, when k = + ∞

$$\frac{1}{1+\exp\left(-k\ln\left(\frac{e(1)}{e(2)}\right)\right)}$$

is equivalent to

$\frac{1}{2}+\frac{1}{2}\,\mathrm{sgn}\left(\frac{e(1)}{e(2)}-1\right)$

i.e., the coefficient-combining strategy proposed in reduces into the choose-max scheme.

2) When k = 0, , the proposed algorithm is equivalent to the average fusion method.

3) For the same (e(1) / e(2)): If larger k is applied, the coefficient combining strategy plays the selection role. However, if smaller k is applied, the coefficient-combining strategy plays the average or smoothing function.

4) The same k : When e(1) is even larger or smaller thane (2), the coefficient- combining scheme plays the selection role. However, when e(1) is closer to e(2), the coefficient-combining strategy plays the weighted average role.

**IMAGE QUALITY METRICS**

Performance Evaluation of the Proposed Fusion Algorithm:

Quality is a characteristic that measures perceived image degradation i.e., in comparison with ideal or perfect image. Evaluation forms an essential part in the development of image fusion techniques. It involves Full Reference where quality is measured in comparison with ideal image and No Reference Methods, which have no reference image. Here we employ Full reference Methods and no reference method[10].

*Full Reference Methods*
Assumptions made in the following equations are as A is the image which is perfect, B is the resultant image. i, j is the pixel row and column index.

*Peak Signal to Noise Ratio (PSNR)*
PSNR is the ratio between the maximum possible power of a signal and the power of corrupting noise that affects the fidelity of its representation. The PSNR measure is given by[11]:-

$$\text{PSNR(db)} = 20\ log\frac{255\sqrt{3MN}}{\sqrt{\sum_{i=1}^{M}\ \ \sum_{j=1}^{N}(B'(i,j)-B(i,j))^2}}$$

Where, $B$ - the perfect image, $B'$ - the fused image to be assessed, $i$ – pixel row index, $j$ – Pixel column index, M, N- No. of row and column.

*Entropy(EN)*
Entropy is an index to evaluate the information quantity contained in an image. If the value of entropy becomes higher after fusing, it indicates that the information increases and the fusion performances are improved.
Entropy is defined as[10]:-

$$\text{E} = \sum_{i=0}^{L-1} p_i \log_2 p_i$$

Where L is the total of grey levels, $p = \{p0, p1, \dots .pL{-}1\}$ is the probability distribution of each level .

*Root Mean Squared Error(RMSE)*
Root Mean square error is measure of image quality index. Larger value of mean square error means that the image is of poor quality. The mathematical equation of RMSE is given as[11]:

$$\text{RMSE} = \sqrt{\frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n}(A_{ij} - B_{ij})^2}$$

Where, $A$ - the perfect image, $B$ - the fused image to be assessed, $i$ – pixel row index, $j$ – pixel column index, m, n- No. of row and column .

*No Reference Method*
Fusion performance can be measured by the following fusion quality evaluation metrics' when we have no reference image:

*Spatial Frequency(SF):*
Spatial Frequency indicates the overall activity in the fused image. The SF is computed as[10]:

Row Frequency:

$$\text{RF} = \sqrt{\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=1}^{N-1}[I_f(x,y) - I_f(x,y-1)]^2}$$

Column Frequency

$$\text{CF} = \sqrt{\frac{1}{MN}\sum_{y=0}^{N-1}\sum_{x=1}^{M-1}[I_f(x,y) - I_f(x-1,y)]^2}$$

Spatial Frequency (SF) = $\sqrt{RF^2 + CF^2}$
Higher the SF means better performance.

*Standard Deviation(SD)*

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Standard Deviation measures the contrast in the fused image. Fused image with high contrast would have high standard deviation[10].

$$SD = \sqrt{\frac{1}{MN}\sum_{y=0}^{N-1}\sum_{x=0}^{M-1}[I_f(x,y) - U_{If}]^2}$$

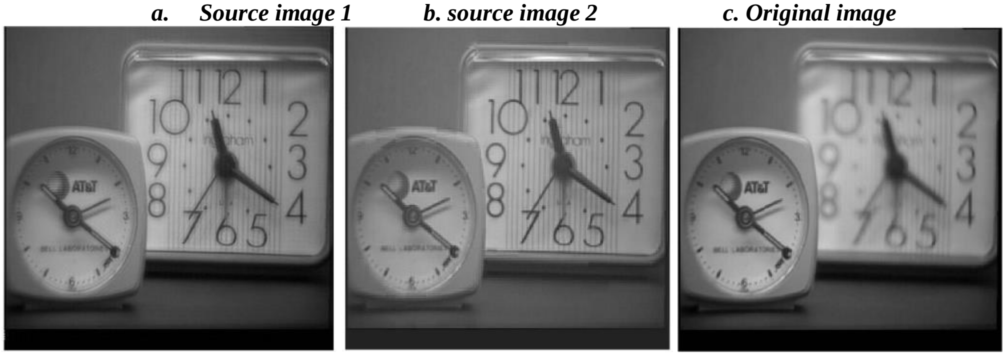
Where the mean is denoted as

$$U_{If} = \frac{1}{MN}\sum_{y=0}^{N-1}\sum_{x=0}^{M-1}\left|I_f(x,y)\right|$$

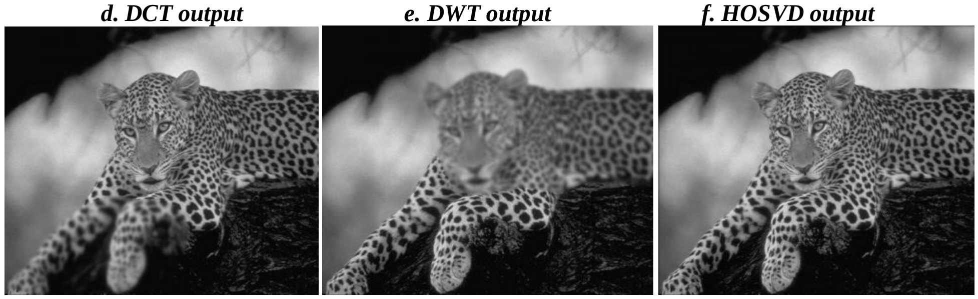
## RESULT AND DISCUSSION

The fusion algorithm developed in this paper is evaluated using the images shown in fig 3. The original images are shown in fig 3(c) and (c1) . The images to be fused are shown in fig 3 (a),(b) which is the image of watch and another set of image shown in fig 3 (a1), (b1) which is the image of tiger.

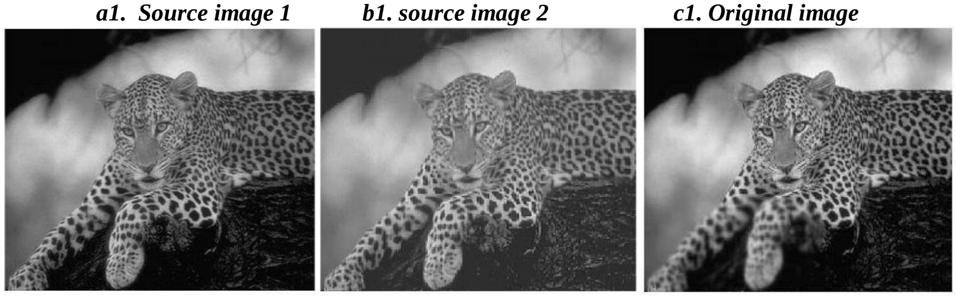
*a. Source image 1* *b. source image 2* *c. Original image*



*d. DCT output* *e. DWT output* *f. HOSVD output*



*a1. Source image 1* *b1. source image 2* *c1. Original image*



*d1. DCT output* *e1. DWT output* *f1. HOSVD output*

***Fig.3 Two pairs of source images and their fused result using different algorithms***

***Table I Comparison of fused result using different metric (source image pair 1)***

| Parameter | Fusion methods | | |
|---|---|---|---|
| | DCT | DWT | HOSVD |
| RMS Error | 109.8018 | 0.064173 | **0.051622** |
| Entropy | 7.3567 | 7.053 | **7.2939** |
| correlation coefficient | 0.99374 | 0.991 | 0.97443 |
| PSNR | 7.3186 | 71.9837 | **73.8741** |
| stddev | 49.5248 | 52.0456 | **50.0478** |
| Mf | 1.6482e-004 | 11.9528 | 8.7017 |

***Table II Comparison of fused result using different metric (source image pair 2)***

| Parameter | Fusion methods | | |
|---|---|---|---|
| | DCT | DWT | HOSVD |
| RMS Error | 112.2919 | 0.098864 | **0.043194** |
| Entropy | 7.4475 | 7.2116 | **7.4531** |
| correlation coefficient | 0.99688 | 0.99177 | 0.99214 |
| PSNR | 7.1238 | 68.2301 | **75.4224** |
| stddev | 64.6692 | 36.6748 | **64.8048** |
| Mf | 3.7443e-004 | 43.9709 | 681.3330 |

These images are out of focus ie. far focused and near focused images of watch and tiger. These images used as source images for different fusion algorithms. Also, fig 3 shows the fused result from DCT, DWT, and HOSVD. Result shows that all methods gives final fused image clear and more informative. But, we can not judge which method gives better result. For this , here employ some quality metrics to identify good one.

These quality metrics which mentioned above are evaluated and listed in Table I , Table II.

From these table, it is shown that the fused result using proposed algorithm are of better entropy(EN), Spatial, standard deviation (SD), Peak Signal to Noise Ratio (PSNR**)**, Root Mean Square Error (RMSE). From these table , it is proved that HOSVD is the best approach for image fusion. The main computational complexity is the multiplications involved in the HOSVD of the divided subtensors. The experiment show that the proposed algorithm is an alternative image fusion approach.

## CONCLUSION

This paper studies the different image fusion methods and proposes a novel HOSVD based image fusion algorithm. The success of the proposed algorithm lies in the following: 1) HOSVD, a fully data-driven technique, is an efficient tool for high-dimensional data decomposition and feature extraction; 2) the SAVC is a feasible activity-level measurement for evaluating the quality of image patches; and 3) the sigmoid-function-based coefficient-combining strategy incorporates the conventional choose-max strategy and the weighted average strategy and thus adapts to different activity levels.

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